Chapter 5.1, 5.3, & 5.4 Graphing

General Graphing

#1 = Find $f(x)$
$f'(x)$
$f''(x)$

#2 = Enter the equations into calculator
$f \rightarrow y_1$
$f' \rightarrow y_2$
$f'' \rightarrow y_3$

#3 $f(x)$ tells us about the original graph, so points on the graph $\Longrightarrow f(x)$

#4 $f'(x)$ = 1st derivative which tells us about
$\Longrightarrow$ Increasing/decreasing
$\Longrightarrow$ mins/maximums

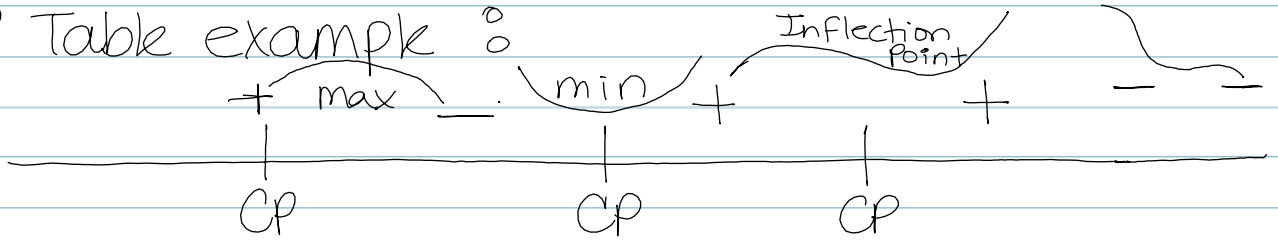
All points together are called Critical Points

Issues: $f'(x) = \emptyset$ (stationary points)

$f'(x)$ = undefined (singular points)

(Potential mins/max until tested)

f' Table example:



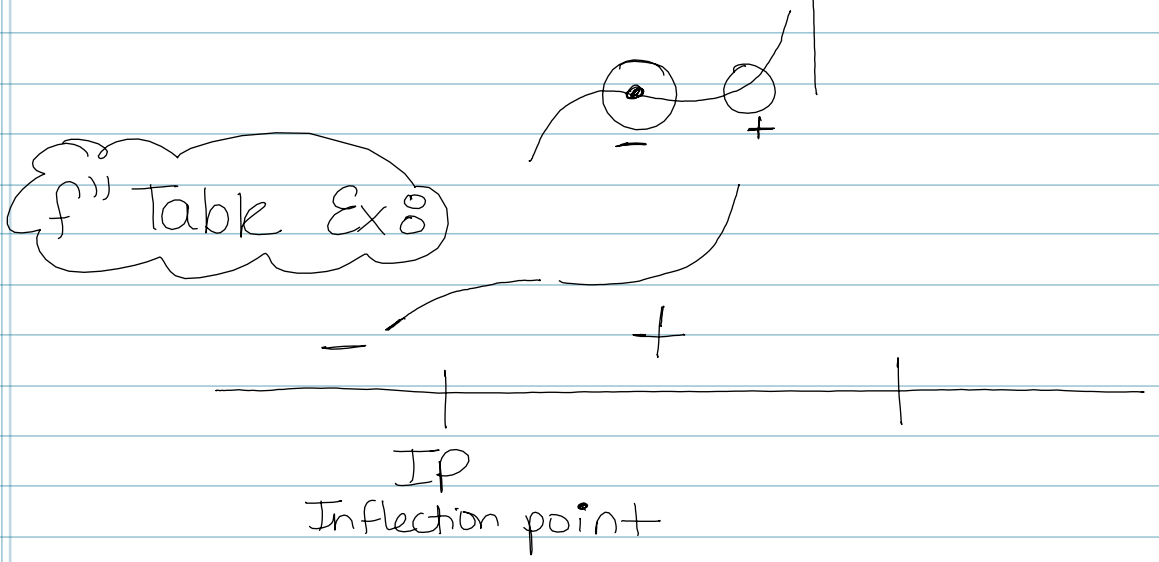
#5 Second Derivative = $f''(x)$

Concavity ∪ ∩

∪ Concave up ∩ Concave down

usually a piece of parabola

$f''(x) = \emptyset$ Potential inflection points | $f''(x) =$ undefined Potential inflection points



#6 Find the points that correspond to important points on the graph (by using the original $f(x)$).

I.e. mins, maxs, IP

#7 Find any intercepts

#8 Draw the graph

NOTE: Just b/c $f'(x)$ or $f''(x)$ is undefined, does not mean that the whole $f(x)$ equation is undefined.

Example: #24 pg. 383

(24) $g(x) = 3x^4 - 16x^3 + 24x^2 + 1$ (with domain $(-\infty, +\infty)$)

y_1

Important b/c these can be end points

Step 1 $g'(x) = 12x^3 - 48x^2 + 48x$ (y_2)

$g''(x) = 36x^2 - 96x + 48$ (y_3)

Step 2 Put in calculator

Step 3 Make sure table set is on Ask, Auto

Step 4 Check: $x = \emptyset$; $y_1 = 1$; $y_2 = \emptyset$; $y_3 = 48$

Step 5 1st derivative tells us where the tangent line is located!!

Step 3 Do 1st derivative analysis

$f'(x) = \emptyset$ (stationary points)

$12x^3 - 48x^2 + 48x = \emptyset$

GCF: $12x$

$12x(x^2 - 4x + 4)$

$12x(x-2)(x-2) = \emptyset$

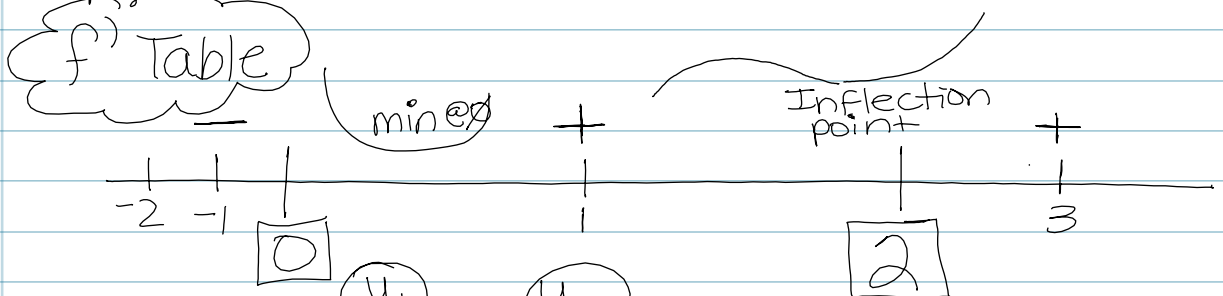
$12x = \emptyset$ | $x = \emptyset$ $\quad$ $x - 2 = \emptyset$ | $x = 2$

2 places you can have min/max

$f'(x)$ = undefined (singular points)

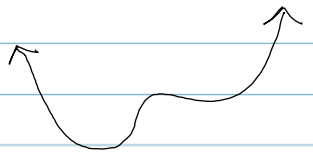
None b/c no denominator. If no denominator it is not possible to have undefined b/c undefined is when the denominator is $= \emptyset$.

Step 4 Draw line + make table (number line)



	y_1	y_2	Signs
$f'(-1) =$	44	−108	(−)
$f'(1) =$	12	+12	(+)
$f'(3) =$	28	+36	(+)

(28: original equation $f(x)$)

Graph kind of looks like  based on my line above.

Step 5 Where does the graph increase?

$(\emptyset, 2) \cup (2, \infty)$ or $(\emptyset, \infty)$

↑ not increasing @ $\emptyset$ so reason why we use ( ) and not [ ]

Step 6 2nd Derivative Analysis
*Tells me about concavity

$f''(x) = \emptyset$

$36x^2 - 96x + 48 = \emptyset$

$12(3x^2 - 8x + 4) = \emptyset$

$12(3x-2)(x-2) = \emptyset$

$3x - 2 = \emptyset$ $x = \frac{2}{3}$

$x - 2 = \emptyset$ $x = 2$

Inflection points

$12 = \emptyset$ *can't use b/c 12 will never $= \emptyset$

$f''(x)$ = undefined

NONE

(b/c no denominator)

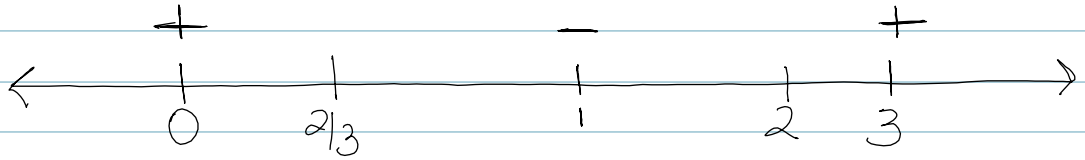
IDEA NOTE:

Draw a box where you keep all your points

Step 7 f'' on Y_3 Table

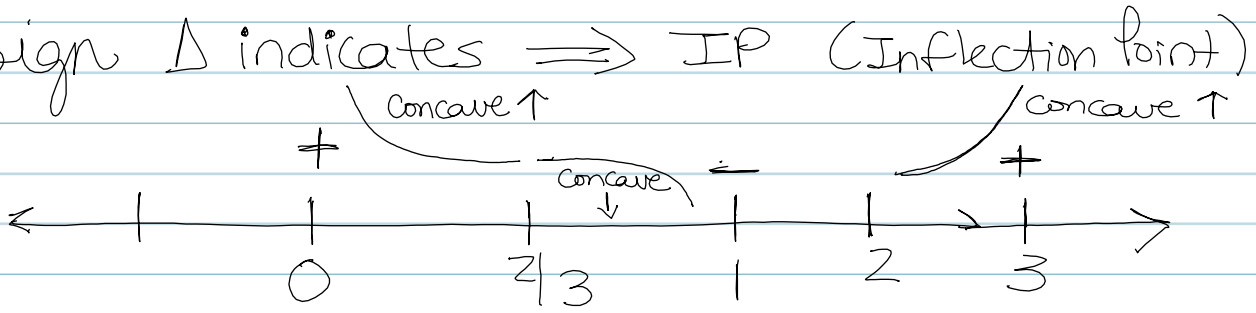
Concave up? $(-\infty, 2/3) \cup (2, \infty)$ @ (+)

Concave down? $(2/3, 3)$ @ (−)



$f''(0) = 48$ (+) $f''(1) = -12$ (−) $f''(3) = 84$ (+)

Sign Δ indicates ⟹ IP (Inflection point)



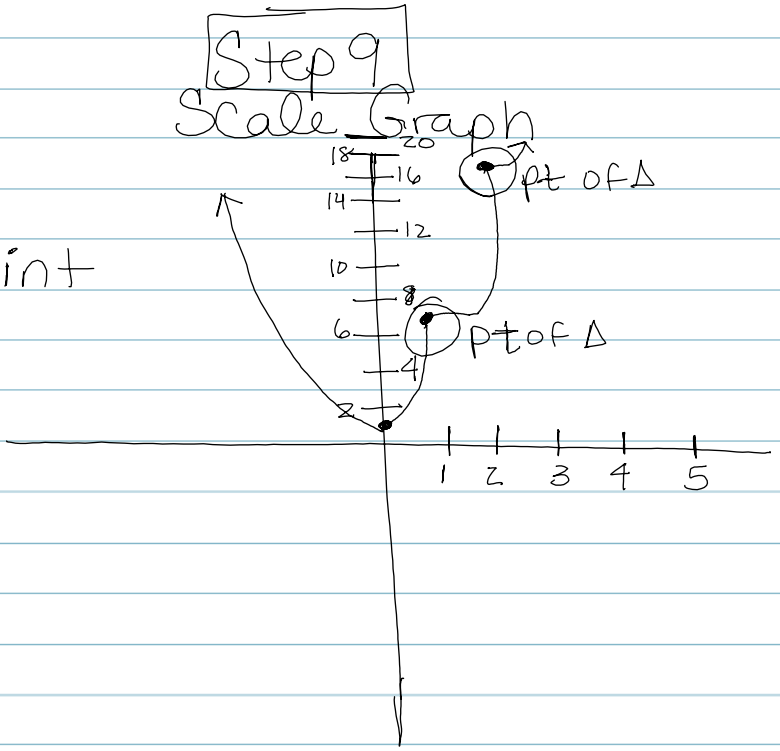
Step 8 Put it all together to make the graph and create a picture.

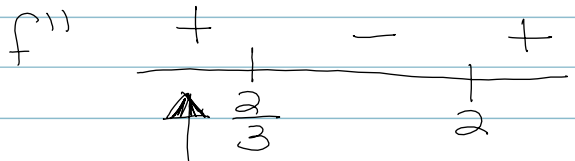
*Go back to box of points; ordered pair table

(f) y_1

x	y	
min 0	1	← y-int
Inflection point (IP) 2	17 (Flat)	
IP 2/3	7.52 (Flat)	

Step 9
Scale Graph





tells me from $-\infty$ to $2/3$ is concave ↑
Compare to graph on previous page

NOTE: Absolute Extrema means looking for max and mins.